5p.

N64-14900 *

CODE-1

CR-55334

Report (NAS 585-4)

31 May, 1962 5p

Memorandum Report

~~on~~

Space Charge Created by an Antenna

in an Ionized Medium

by

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(NASA CR-55334; NAS-585-4; OTS:

OTS PRICE

XEROX $ 1.10 ph.

MICROFILM $ 0.80 mf.

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Prepared for

National Aeronautics and Space Administration

under

(NASA Contract NAS 5-585)

ELECTROMAGNETIC RESEARCH CORP~~ORATION~~

5001 ~~COLLEGE~~ AVENUE

COLLEGE PARK, MD.

2920254

<u>Space Charge Created by an Antenna in an Ionized Medium</u>

The field in the neighborhood of a radiating electric dipole antenna contains a radial component of electric field. The magnitude of this field varies as $1/r$ (r = distance from antenna axis) in the region close to the axis and not too close to the ends of the antenna. If the medium contains free electrons, these will oscillate with a radial component of vibration, which will be non-linear because of the variation of the radial electric field strength with radial distance of the electron. The effects of this non-linear motion on the resulting distribution of ionization in the vicinity of the antenna will be considered here.

The procedure that will be used is essentially a perturbation analysis, since it will be assumed that the electric field due to the r-f voltage applied to the antenna still varies as $1/r$. Consequently, the results obtained should be considered as a first approximation to the actual state of affairs.

In order to combine the analysis to the essential effects, collisions and the earth's magnetic field will be neglected at first.

The radial component of electron motion then obeys the differential equation

$$m\ddot{r} = eE_r = \frac{eE_0}{r} \cos\omega t$$

where E_0 is the radial electric field at unit radial distance and e is the charge of an electron (negative). This equation may be rewritten as

$$r'' = \frac{\alpha}{r} \cos\tau \qquad (1)$$

where

$$t = \omega\tau$$

$$\alpha = eE_0/(\omega^2 m)$$

$$r'' = d^2r/d\tau^2$$

The zero-order solution of (1), obtained by replacing r on the right-hand-side by an average value r_0, is

$$r = r_0 - \frac{\alpha}{r_0} \cos \tau \qquad (2)$$

Comparison of (1) and (2) reveals that the acceleration and position of the electron have opposite phases (which is obvious from physical considerations). Thus the maximum repulsive force occurs at the nearest distance of the electron from the antenna axis, and vice versa. Hence it follows from (1), because of the $1/r$ factor on the right, that the average repulsive force is greater than the average attractive force over an r-f cycle. This residual repulsion per cycle is

$$\bar{F} = eE_0 \int_0^{2\pi} \frac{\cos \tau}{r} \, d\tau$$

Inserting for r the value given in (2), we obtain

$$\bar{F} = eE_0 \int_0^{2\pi} \frac{\cos \tau}{r_0 - \frac{\alpha}{r_0} \cos \tau} \, d\tau \approx \left(\frac{eE_0}{\omega}\right)^2 \frac{1}{2mr_0^3}$$

This is equivalent to a static electric field

$$\bar{E} = \frac{\bar{F}}{e} = \frac{eE_0^2}{2m\omega^2 r_0^3}$$

For equilibrium, this static electric field must be balanced by an opposite field due to the combined effects of space charge in the medium and a possible charge induced on the antenna.

In the immediate vicinity of the antenna, within a distance α/R_0 from the surface (R_0 = antenna radius), the electrons will strike the antenna. If we assume that these are collected by the antenna, then there will be created a positive ion sheath of thickness $d = |\alpha|/R_0$. Outside this sheath there will be an inhomogeneous mixture of electrons and positive ions, the electron density being somewhat greater than the positive ion density because of the outward force F on the electrons. We now solve for the electron distribution, and the charge collected on the antenna.

We assume a homogeneous distribution of positive ions of density N_i in the medium, and also that a charge Q_0 per unit length collects on the antenna. We denote the electron density by N (a function of r) and the differential charge density by δN,

$$\delta N = N - N_i .$$

The total field in the medium then will be the sum of four components:

(a) The field $\bar{\bar{E}}$ given by (3);

(b) the field due to the charge on the antenna;

(c) the field due to the positive ion sheath;

(d) the field due to the electron-positive ion mixture outside the sheath.

The field due to the charge on the antenna will be

$$E_a = \frac{Q_o}{2\pi\varepsilon_o r} \tag{4}$$

The field due to the space charge in the medium, represented by δN, is

$$E_s = \frac{\Delta Q}{2\pi\varepsilon_o r} = \frac{e}{\varepsilon_o r}\int_{R_o+\alpha/R_o}^{r} r\delta N dr \tag{5}$$

The field due to the positive ion sheath is

$$E_p = \frac{2\pi R_o\alpha/R_o N_i\,|e|}{2\pi\varepsilon_o r} = -\frac{\alpha N_i e}{\varepsilon_o r}. \tag{6}$$

For equilibrium, the total electrostatic field in the medium must be zero. Consequently,

$$\bar{\bar{E}} + E_a + E_s + E_p = 0.$$

Replacing r_o in (3) by r, this gives us the relation

$$\int_{R_o+\alpha/R_o}^{r} r\delta N dr = \alpha N_i - \frac{Q_o}{2\pi e} - \frac{\varepsilon_o E_o^2}{2m\omega^2 r^2} \tag{7}$$

By differentiation, this gives

$$\delta N = \frac{\varepsilon_o E_o^2}{m\omega^2 r^4} \tag{8}$$

With this value of N, we obtain from (7) with $r = \infty$,

$$Q_o = 2\pi e\left(\alpha N_i - \frac{\varepsilon_o E_o^2}{2m\omega^2 R_o^2}\right) = \frac{2\pi e E_o}{m\omega^2}\left(e - \frac{\varepsilon_o E_o}{2R_o^2}\right) \tag{9}$$

This charge per unit length usually will be negative in practical cases, since the first term in the parentheses usually will be negligible relative to the second. This means that the field of the positive ion sheath is negligible compared to the field due to the charge on the antenna.

From (8), the differential charge density decreases as the fourth power of the distance from the antenna axis, the charge in a narrow circular ring of radius r decreasing as $1/r^3$. Thus the inhomogeneity is confined to a small region very close to the antenna. δN also is independent of the actual average density N_i, for a given value of E_o. However, for a given current in the antenna, E_o depends on the dielectric constant of the medium, which is a function of the electron density. In fact, E_o contains a factor of $1/n$, where n is the refractive index of the medium. For an ionized medium

$$n^2 = 1 - X$$

where

$$X = \omega_p^2/\omega^2.$$

Since (8) contains E_o^2, δN will contain a factor

$$1/n^2 = (1 - X)^{-1}$$

for a constant antenna current.

In summary, the above analysis gives the following results:

A positive ion sheath is produced around the antenna, its thickness being

$$d = |\alpha|/R_o = \frac{|e|E}{m\omega^2 R_o}$$

The antenna collects a negative charge per unit length given by

$$Q_o = \frac{\pi e E_o E_o^2}{m\omega^2 R_o^2} = -\frac{\pi \varepsilon_o E_o d}{R_o}$$

The electron density exceeds the ion density in the region outside the ion sheath by an amount

$$\delta N = \frac{\varepsilon_o E_o^2}{m\omega^2 r^4}.$$